Time value of money

OR

simple & compound interest with applications of simple annuity

- Interest is as good as Rent to be paid by the borrower for using lender's money.

simple interest

compound interest

- In a commercial transaction why interest is paid ?

⟹
1. Time value of money
2. Inflation (Rise in General price level)
3. opportunity cost
4. Liquidity prefrence
5. Risk factor

Amount = principle amt + Interest

principle = Amount − Interest

Interest = Amount − principle

- Mr. A deposits ₹ 20,00,000 in BOM, After 10 years he get ₹ 65,00,000 from Bank then

₹20,00,000 = principle amount = P

₹65,00,000 = Amount = P + I

₹65,00,000 − ₹20,00,000 = ₹45,00,000 = Interest

Interest

Simple Interest

Here interest is always calculated on original principle amount invested

Compound Interest

Here interest is calculated on Revised principle amount.

Revised principle amount = (original principle invested) + (Accumulated Interest)

. Mr. A invested ₹20,000 in a Bank for 5 years @ 10% P.a.S.I. Find amount receivable from Bank at the end of 5 years ?

⟹

Year	opening Balance (₹)	Interest (simple) (₹)	closing Balance (₹)
1	20,000	20,000 x 10% = 2,000	22,000
2	22,000	20,000 x 10% = 2,000	24,000
3	24,000	20,000 x 10% = 2,000	26,000
4	26,000	20,000 x 10% = 2,000	28,000
5	28,000	20,000 x 10% = 2,000	30,000

Amount receivable at the end of 5 years = ₹30,000

- principle amt invested = ₹20,000
- simple Interest = ₹10,000

$$10,000 = 20,000 \times 10\% \times S$$

$$10,000 = 20,000 \times S \times \left(\frac{10}{100}\right)$$

Simple Interest $= P \times n \times r$

Amount = principle amt + simple interest

$$A = P + Pnr$$

$$A = P(1+nr)$$

- Mr. X invested ₹ 50,00,000 @ 12% p.a. S.I. for 12 years. Find A, SI.

$\Longrightarrow$

$$SI = P.n.r$$
$$= 50,00,000 \times 12 \times 12\%$$
$$= 72,00,000$$

$$\text{Amount} = P(1+nr)$$
$$= 50,00,000\,[1 + 12(0.12)]$$
$$= 50,00,000\,(1 + 1.44)$$
$$= 50,00,000 \times 2.44$$
$$= ₹\ 1,22,00,000/-$$

- Mr. A invested ₹20,000 in a Bank for 5 years @ 10% P.a.C.I. Find amount receivable from Bank at the end of 5 years ?

$\Longrightarrow$

Year	opening Balance (₹)	Interest (compound) (₹)	closing Balance (₹)
1	20,000	20,000 x 10% = 2,000	22,000
2	22,000	22,000 x 10% = 2,200	24,200
3	24,200	24,200 x 10% = 2,420	26,620
4	26,620	26,620 x 10% = 2,662	29,282
5	29,282	29,282 x 10% = 2,928.20	32,210.20

Amount receivable at the end of 5 years = ₹32,210.20

principle amt invested = ₹20,000

compound interest = ₹12,210.20

- $32,210.20 = 20,000 \times 1.10 \times 1.10 \times 1.10 \times 1.10 \times 1.10$

$$32,210.20 = 20,000 \times (1.10)^5$$

$$32,210.20 = 20,000 \times (1 + 0.10)^5$$

- $A = P \cdot (1+r)^n$

- $\left(\begin{matrix}\text{compound} \\ \text{interest}\end{matrix}\right) = \text{Amount} - \text{principle}$

$$= P(1+r)^n - P$$

$$= P\left[(1+r)^n - 1\right]$$

simple →	SI = P.n.r	CI = P[(1+r)^n − 1]	← compound
	Amount = A	Amount = A	
	= P(1+nr)	= P(1+r)^n	
	where P = principle n = No. of years		
	r = Rate of interest p.a.		

- Mr. x invested ₹ 5,00,000 in a Bank @ 11.50% p.a.C.I. for 38 years. Find A, CI.

$\Longrightarrow$

① $A = P(1+r)^n$

$= 5,00,000\ (1+0.1150)^{38}$

$= 5,00,000 \times (1.1150)^{38}$

$= 5,00,000 \times 62.581354768 8$

$= 3,12,90,677/-$

$1.1268^{70} =$	4258.7160 2502
$1.096366^{79} =$	1433.8019 1395
$1.18562289^{58} =$	19,449.1842617
$1.10^{61} =$	334.92980347

- P = 50,12,890, r = 13.90% P.a.S.I.
n = 25 years, A = ? SI = ?

$\Longrightarrow$ $SI = pnr$

$= 50,12,890 \times 25 \times 13.90\% = 1,74,19,792.75$

Amount = 1,74,19,792.75 + 50,12,890

= 2,24,32,682.75

(OR) $A = p(1+nr) = 50,12,890 \times [1 + (25 \times 0.1390)]$

$= 50,12,890 \times 4.475$

$= 2,24,32,682.75$

simple	wmpound
$SI = P.n.r$	$CI = P\left[(1+r)^n - 1\right]$
$A = P(1+nr)$	$A = P(1+r)^n$

(1) Mr. A invested ₹ 5,00,00,000 for 35 years @ 14.50% P.a.C.I. Find Amount, CI.

$$A = P(1+r)^n$$
$$= 5,00,00,000 \times (1+0.1450)^{35}$$
$$= 5,00,00,000 \times (1.1450)^{35}$$
$$= 571,69,18,987$$

Compound Interest = 571,69,18,987 − 5,00,00,000
= 566,69,18,987

(2) (i) $P = 50,00,000$
$r = 8.50\%$ P.a.C.I.
$n = 25$ years
$A = ?$

$$A = P(1+r)^n$$
$$= 50,00,000\,(1.0850)^{25}$$
$$= 3,84,33,812/-$$

(ii) $P = 80,000$, $n = 10$ years, $r = 12\%$ P.a.S.I.
$A = ?$

$$A = P(1+nr)$$
$$= 80,000\,[1 + (10 \times 0.12)]$$
$$= 1,76,000/-$$

(3) $P = 80,000$, $n = 10$ years, $r = 12.50\%$ P.a.S.I.

$SI = ?$, $A = ?$

$\Rightarrow$ Simple Interest $= P \cdot n \cdot r$

$= 80,000 \times 10 \times 12.50\%$

$= 1,00,000$

$A =$ P + simple interest

$= 80,000 + 1,00,000$

$= 1,80,000$

OR $A = P(1+nr)$

$= 80,00 \left[1 + (10 \times 0.1250)\right]$

$= 1,80,000$

(4) Mahesh invested a sum at 12% P.a.C.I. for 20 years. He gets ₹ 50,12,000 at the end. Find sum invested?

$\Rightarrow$ $r = 12\%$ P.a.C.I, $n = 20$ years, $A = 50,12,000$

$P = ?$

$A = P(1+r)^n$

$50,12,000 = P(1.12)^{20}$

$50,12,000 = P \times 9.64629\,309311$ $\quad \therefore P = 5,19,578$

(5) Ram invested ₹ 50,00,000 at r% P.a.C.I. for 16 years. He gets ₹ 2,50,00,000 at the end. Find 'r'?

$\Rightarrow$ $P = 50,00,000$ $n = 16$ years $A = 2,50,00,000$

$A = P(1+r)^n$

$2,50,00,000 = 50,00,000\,(1+r)^{16}$

$(1+r)^{16} = 5$

$\left[(1+r)^{16}\right]^{1/16} = 5^{1/16}$

$1+r = 1.105823$

$r = 0.105823$ i.e. 10.5823% P.a.C.I.

⑥ Shyam invested ₹ 80,000 for 20 years @ 15% p.a.s.I. Find

ⓐ Interest earned by shyam?

ⓑ Amount receivable at the end?

$\Rightarrow$ Simple Interest $= P.n.r$

$= 80,000 \times 20 \times 15\%$

$= ₹\ 2,40,000/-$

Amount = principle + simple interest

$= ₹80,000 + ₹2,40,000$

$= ₹3,20,000/-$

⑦ Rahul invested ₹ 50,000 in a Bank @ 10% p.a.c.I. for 6 years. Find Interest amount, Amount receivable at the end?

$\Rightarrow$

Year	opening Balance	compound interest	closing Balance (₹)
1	50,000	5,000	55,000
2	55,000	5,500	60,500
3	60,500	6,050	66,550
4	66,550	6,655	73,205
5	73,205	7,320.50	80,525.50
6	80,525.50	8,052.55	88,578.05
		Total = 38,578.05	

$[(50,000 \times 1.10) \times 1.10] \times 1.10 \times 1.10 \times 1.10 \times 1.10 = 88,578.05$

$50,000 \times (1.10)^6 = 88,578.05$

$50,000 \times (1+0.10)^6 = 88,578.05$

- $P(1+r)^n = A$

$\therefore$ CI = ₹ 38,578.05

- $CI = A - P = P(1+r)^n - P$

$= P[(1+r)^n - 1]$

(8) $A = 9,00,000$, $P = ?$, $r = 12\%$ P.a.S.I.

$n = 8$ years 3 months

$\Rightarrow A = P(1+nr)$

$9,00,000 = P\left[1 + (8.25 \times 0.12)\right]$

$9,00,000 = P \times 1.99$

$\boxed{P = 4,52,261}$

(9) $A = 9,00,000$, $P = 60,000$, $r = \_\_\_\_\_\ \%$ P.a.C.I.

$n = 10$ years

$\Rightarrow A = P(1+r)^n$

$9,00,000 = 60,000\ (1+r)^{10}$

$(1+r)^{10} = 15$

$1+r = 15^{1/10}$

$r = 31.1125\%$ P.a.C.I.

(10) $A = 15,00,000$, $P = 80,000$, $r = \_\_\_\_\_\ \%$ P.a.C.I.

$n = 14$ years

$\Rightarrow A = P(1+r)^n$

$15,00,000 = 80,000 \times (1+r)^{14}$

$(1+r)^{14} = 18.75$

$(1+r) = 18.75^{1/14}$

$r = 23.2988\%$ P.a.C.I.

(11) $A = 18,00,000$, $P = 35,000$, $r = \_\_\_\_\_\ \%$ P.a.S.I.

$n = 58$ years

$\Rightarrow A = P(1+nr)$

$18,00,000 = 35,000\ (1+58r)$

$r = 86.9458\%$ P.a.S.I.

(12) $P = 90,000$, $A = 12,00,000$, $r = \_\_\_\_$ % P.a.S.I.,
$n = 14$ years 9 months $SI = P \cdot n \cdot r$

$\Rightarrow A = P(1+nr)$ (OR) $11,10,000 = 90,000 \times 14.75 \times r$

$12,00,000 = 90,000(1 + 14.75r)$

$r = 83.6158\%$ P.a.S.I. $r = 83.6158\%$

(13) $P = ?$, $A = 90,12,500$, $r = 16.13\%$ P.a.C.I,
$n = 10$ years

$\Rightarrow A = P(1+r)^n$

$90,12,500 = P(1.1613)^{10}$

$P = 20,20,231/-$

(14) $P = ?$, $A = 80,000$, $r = 12.50\%$ P.a.S.I.
$n = 20$ years

$\Rightarrow A = P(1+nr)$

$80,000 = P[1 + (20 \times 0.1250)]$

$\boxed{P = 22,857}$

(15) $P = ?$, $r = 15\%$ P.a.C I., $A = 8,000$
$n = 5$ years

$\Rightarrow A = P(1+r)^n$

$8,000 = P(1.15)^5$

$\boxed{P = 3,977}$

(16) $P = 60,000$, $A = 1,00,000$, $r = 10\%$ P.a.S.I.
$n = \_\_\_\_$ years

$\Rightarrow A = P(1+nr)$

$1,00,000 = 60,000(1 + n \times 0.10)$

$1.666666 = 1 + 0.10n$

$0.10n = 0.6666666$

$n = 6.66666666$ years

$n = 6\frac{2}{3}$ years = 6 years & 8 months

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(17) $A = 2,50,000$, $P = 50,000$, $r = 10\%$ P.a.s.I.

$n =$ ____ years

$\Rightarrow A = P(1+nr)$

$2,50,000 = 50,000(1 + n \times 0.10)$

$n = 40$ years

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(18) $A = 2,50,000$, $P = 60,000$, $r =$ ____ % P.a.c.I.

$n = 18$ years

$\Rightarrow A = P(1+r)^n$

$2,50,000 = 60,000(1+r)^{18}$

$(1+r)^{18} = 4.16666666$

$1+r = (4.166666666)^{1/18}$

$r = 8.2526\%$ P.a.c.I.

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(19) $A = 90,000$, $P = 8,000$, $r = 12\%$ P.a.c.I.

$n =$ ____ years

$\Rightarrow A = P(1+r)^n$

$90,000 = 8,000(1.12)^n$

$(1.12)^n = 11.25$

Taking Log on both sides

$\text{Log}(1.12)^n = \text{Log}\,11.25$

$n \times \text{Log}\,1.12 = \text{Log}\,11.25$

$n = \frac{\text{Log}\,11.25}{\text{Log}\,1.12}$

$n = \frac{1.05119132306}{0.04921792957}$

$n = 21.35789$ years

(20) $P = 90{,}000$, $A = 27{,}00{,}000$, $n = \_\_\_$ years,
$r = 10.50\%$ P.a. c.I.

$\Rightarrow$ $A = P(1+r)^n$

$27{,}00{,}000 = 90{,}000(1.1050)^n$

$\therefore 1.1050^n = 30$

Taking Log on both sides

$n \cdot \text{Log}\, 1.1050 = \text{Log}\, 30$

$$n = \frac{\text{Log}\, 30}{\text{Log}\, 1.1050} = \frac{1.47719}{0.043362315} = 34.07 \text{ years}$$

(21) A sum invested at simple interest becomes double in 10 years. How many years it will take so that sum becomes triple of sum invested?

(a) 15 years (b) 20 years (c) 30 years (d) None

$\Rightarrow$

$A = P(1+nr)$

$2P = P(1+10r)$

$2 = 1+10r$

$10r = 1$

$r = 0.10 = 10\%$ P.a.S.I

$A = 3P$, $n = ?$, $r = 10\%$ P.a.S.I

$A = P(1+nr)$

$3P = P(1+n \times 0.10)$

$3 = 1 + 0.10n$

$0.10n = 2$

$n = 20$ years

Simple Interest $= P \times n \times r$

Compound Interest $= P\left[(1+r)^n - 1\right]$

Amount when interest in simple $= P\left[1+nr\right]$

Amount when interest is compound $= P\left[(1+r)^n\right]$

① $P = 95{,}000$, $r = 15\%$ p.a.c.I., $n = 18$ years

$CI = ?$, $A = ?$

$\Rightarrow$

$$A = P(1+r)^n$$
$$= 95{,}000(1.15)^{18}$$
$$= 11{,}75{,}668/-$$

compound interest $= 11{,}75{,}668 - 95{,}000$
$= 10{,}80{,}668/-$

② $P = 1{,}00{,}000$, $A = 12{,}50{,}000$, $r = \_\_\_\_\_ \%$ p.a.c.I.

$n = 16$ years

$\Rightarrow$

$$A = P(1+r)^n$$
$$12{,}50{,}000 = 1{,}00{,}000(1+r)^{16}$$
$$(1+r)^{16} = 12.50$$
$$(1+r) = 12.50^{1/16}$$
$$r = 17.09999\%$$

③ $P = 15{,}00{,}000$, $A = 50{,}00{,}000$, $r = \_\_\_\_\_ \%$ p.a.s.I.

$n = 40$ years

$\Rightarrow$

$$A = P(1+nr)$$
$$50{,}00{,}000 = 15{,}00{,}000(1+40r)$$
$$r = 5.8333333\% \text{ p.a.s.I.}$$

④ $P = 17{,}500$, $r = 8.50\%$ p.a.s.I., $n = 9$ years 9 months

$SI = ?$, $A = ?$

$\Rightarrow$

simple Interest $= P \cdot n \cdot r$
$= 17{,}500 \times 9.75 \times 8.50\%$
$= 14{,}503.125$

Amount $= 32{,}003.125$

⑤ $P = 65,000$, $A = 2,00,000$, $r = \_\_\_\_$ % P.a.c.I.

$n = 22$ years

$\Rightarrow$ $A = P(1+r)^n$

$2,00,000 = 65,000\,(1+r)^{22}$

$(1+r)^{22} = 3.07692307692$

$1+r = \left(3.07692307692\right)^{1/22}$

$r = 5.2422\ \%$ P.a.c.I.

⑥ $P = 50$, $r = 22.50\%$ P.a.c.I., $n = 28$ years

$A = ?$, $CI = ?$

$\Rightarrow$ $A = P(1+r)^n$

$= 50\,(1.2250)^{28}$

$= 14,682/-$

$CI = 14,682 - 50$

$= 14,632/-$

⑦ A sum invested at compound interest becomes triple in 10 years. Find rate of interest?

$\Rightarrow$ $A = P(1+r)^n$

$3P = P(1+r)^{10}$

$(1+r)^{10} = 3$

$1+r = 3^{1/10}$

$r = 11.6137\%$

⑧ A sum invested at compound interest becomes double in 7 years. How many years it will take so that sum becomes 128 times of sum invested?

$\Rightarrow$

$A = P(1+r)^n$

$2P = P(1+r)^7$

$(1+r)^7 = 2$

sum invested	After ___ Years						
	7	14	21	28	35	42	49
P	2P	4P	8P	16P	32P	64P	128P

$A = 128P$, $n = ?$

$A = P(1+r)^n$

$128\cancel{P} = \cancel{P}(1+r)^n$

$\therefore (1+r)^n = 128 = 2^7$

$(1+r)^n = [(1+r)^7]^7$

$(1+r)^n = (1+r)^{49}$

$\therefore n = 49$ years

(9) A sum invested at simple interest becomes 3 times in 8 years. Find how many times it becomes in 48 years?

$\Rightarrow$

$A = P(1+nr)$

$3\cancel{P} = \cancel{P}(1+8r)$

$2 = 8r$

$r = 25\%$

sum invested	After ___ Years					
	8	16	24	32	40	48
P	3P	5P	7P	9P	11P	13P

$A = P(1+nr)$

$= P[1+(48 \times 0.25)]$

$= 13P$

13 times of sum invested

(10) Short-cut

simple interest ↙ ↘ compound interest

After — years

	x	2x	3x	4x	5x
P	2P	3P	4P	5P	6P
P	5P	9P	13P	17P	21P
P	6P	11P	16P	21P	26P

After — years

	x	2x	3x	4x	5x
P	2P	4P	8P	16P	32P
P	3P	9P	27P	81P	243P
P	5P	25P	125P	625P	3125P

(11) $A = ?,\ P = 80{,}000,\ r = 13\%$ P.a.c.I., $n = 20$ years

$$\Rightarrow A = P(1+r)^n$$

$$= 80{,}000\,(1.13)^{20}$$

$$= 9{,}21{,}847/-$$

$$A = P(1+r)^n$$

$$\left(\begin{matrix}\text{Future}\\ \text{value}\end{matrix}\right) = \left(\begin{matrix}\text{present}\\ \text{value}\end{matrix}\right) \times (1+r)^n$$

(12) Find Future value of ₹ 9,00,000 after 35 years @ 8% P.a.

$\Rightarrow$

Future value = present value $\times (1+r)^n$

$= 9{,}00{,}000 \times (1.08)^{35}$

$= ₹\ 1{,}33{,}06{,}810/-$

(13) Find present value of ₹ 1,33,06,810 receivable after 35 years if money is invested @ 8% P.a.c.I.

$\Longrightarrow$

Future value = present value $\times (1+r)^n$	Future value = present value $\times (1+r)^n$
$1,33,06,810 =$ present value $\times (1.08)^{35}$	present value = Future value $\times \frac{1}{(1+r)^n}$
$1,33,06,810 =$ present value $\times 14.7853442937$	Present value = (Future value) $\times$ (Discounting factor)
present value $= 1,33,06,810 \times \frac{1}{14.7853442937}$	
present value $= 1,33,06,810 \times 0.0676345427$	$= 1,33,06,810 \times 0.0676345427$
present value $= 9,00,000/-$	$= 9,00,000/-$

Future value = present value $\times (1+r)^n$

Present value = Future value $\times$ Discounting factor

where,

Discounting factor $= \frac{1}{(1+r)^n}$

- How to find discounting factor on calculator for n^{th} year?

$\Longrightarrow$ $1 \div (1+r)$ then press '=' button till step count comes $(n+2)$

$\Downarrow$

You will find discounting factor on calculator screen.

Find present value of ₹ 35,00,000 receivable after 28 years if money is 12.50% effective ?

$\Rightarrow$

Present value = Future value × Discounting factor

= 35,00,000 × 0.03695977945

= 1,29,359.228075

Find Future value of ₹ 1,29,359.228075 after 28 years @ 12.50% P.A.C.I.

$\Rightarrow$

Future value = present value $\times (1+r)^n$

$= 1,29,359.228075 \times (1.1250)^{28}$

= 35,00,000/-

Find present value of ₹ 90,00,000 receivable after 25 years if money is 18.25% effective ?

$\Rightarrow$

Present value = Future value × Discounting factor

= 90,00,000 × 0.01513461103

= 1,36,211.49927

Find future value of ₹ 1,36,211.49927 after 25 years @ 18.25% p.a.

$\Rightarrow$

Future value = present value $\times (1+r)^n$

$= 1,36,211.49927 \times (1.1825)^{25}$

= 90,00,000/-

① Find present value of bond of ₹1000 receivable after 5 years? (Buyer's expected RoI = 12% p.a.)

$\Rightarrow$

present value = Future value × Discounting factor

= ₹1000 × 0.567426

= ₹567.43

② Find present value of ₹50,00,000 payable after 25 years if money can fetch 14.5025% p.a. compound interest?

$\Rightarrow$ present value = Future value × Discounting factor

= ₹50,00,000 × 0.033855230 48

= ₹1,69,276/-

③ A TV can be purchased by paying ₹20,000 now and 50000, 40000, 80000 at the end of years 1,2,3 resp. Find cash down price of TV if money can fetch 15% interest?

$\Rightarrow$

			Present value
	NOW	20,000	20,000
	1	50,000	50,000 × 0.86956 = 43,478
At the	2	40,000	40,000 × 0.756143 = 30,246
end of	3	80,000	80,000 × 0.657516 = 52,601
Year			

cash down price = ₹ 1,46,325

④ A House can be purchased by paying ₹ 50,00,000 now and ₹ 90,60,30,80 lakhs at the end of years 1,2,3,4 respectively. Find cash down price of House if money can fetch 16.50% interest.

⟹

			Present value
Now		50,00,000	50,00,000
At the	1	90,00,000	77,25,322
end of	2	60,00,000	44,20,785
year	3	30,00,000	18,97,333
	4	80,00,000	43,42,965
		cash down price =	2,33,86,405
		of House	

⑤ A House can be purchased by paying ₹ 40,00,000 now and 5 equal installments of ₹ 20,00,000 payable at the end of each year. Find cash down price of House if money can fetch 18% interest?

⟹

			Present value
	Now	40,00,000	40,00,000
	1	20,00,000	16,94,915
Payable at	2	20,00,000	14,36,369
the end of	3	20,00,000	12,17,262
Year	4	20,00,000	10,31,578
	5	20,00,000	8,74,218
		cash down price of House =	₹ 1,02,54,342

(6) Mr. A invested ₹ 10,000 for 1 year @ 12% p.a.c.q.

Find amount receivable at the end of year 1.

$\Rightarrow$

Quarters	op. Bal	Interest	cl. Bal
1	10,000	$10,000 \times 12\% \times \frac{1}{4} = 300$	10,300
2	10,300	$10,300 \times 3\% = 309$	10,609
3	10,609	$10,609 \times 3\% = 318.27$	10,927.27
4	10,927.27	$10,927.27 \times 3\% = 327.8181$	11,255.0881

(OR)

$P = 10,000$, $r = 12\%$ p.a.c.q, $n = 1$ year $= 4$ quarters

$$A = P(1+r)^n$$

$$= 10,000\left(1 + \frac{0.12}{4}\right)^4$$

$$= 10,000\left(1 + 0.03\right)^4$$

$$= 10,000\ (1.03)^4 = 11,255.0881$$

$A = P(1+r)^n$

A = Amount

P = principle

r = rate of interest for the conversion period

n = No. of conversion periods

⑦ $P = 90,00,000$ $\Longrightarrow$ $A = P(1+r)^n$

$r = 12\%$ p.a.c.m.

$n = 8.25$ years

$A = ?$

$= 90,00,000\left(1+\frac{0.12}{12}\right)^{8.25\times12}$

$= 90,00,000\,(1.01)^{99}$

$= 2,41,02,301/-$

⑧ $P = 95,000$ $\Longrightarrow$ $A = P(1+r)^n$

$r = 13.50\%$ p.a.c.q.

$n = 11$ years 6 months

$A = ?$

$= 95,000\left(1+\frac{0.1350}{4}\right)^{46}$

$= 95,000\,(1.03375)^{46}$

$= 4,37,359/-$

⑨ $P = 60,000$ $\Longrightarrow$ $A = P(1+r)^n$

$r = 18.50\%$ p.a.c.m.

$n = 3$ years 7 months

$A = ?$

$= 60,000\left(1+\frac{0.1850}{12}\right)^{43}$

$= 60,000\,(1.015416666666)^{43}$

$= 1,15,839/-$

compounded	No. of conversion periods in a year
annually	1
semi-annually OR Bi-annually OR Half yearly	2
quarterly	4
monthly	12
weekly	52
Daily	365
Fortnightly	24

① $A = ?$, $P = 4,50,000$, $r = 18\%$ p.a.c.m., $n = 3$ years 10 months

$$\Rightarrow A = P(1+r)^n$$

$$= 4,50,000\left(1+\frac{0.18}{12}\right)^{46}$$

$$= 4,50,000 \times (1.015)^{46}$$

$$= 8,92,587/-$$

② $A = ?$

$P = 98,000$

$r = 26.26\%$ p.a.c.w.

$n = 1.50$ years

$$\Rightarrow A = P(1+r)^n$$

$$= 98,000\left(1+\frac{0.2626}{52}\right)^{78}$$

$$= 98,000\,(1.00505)^{78}$$

$$= 1,45,166/-$$

③ $A = 95,00,000$

$P = ?$

$r = 22.50\%$ p.a.c. semiannually

$n = 7.50$ years

$$\Rightarrow A = P(1+r)^n$$

$$95,00,000 = P\left(1+\frac{0.2250}{2}\right)^{15}$$

$$95,00,000 = P\,(1.1125)^{15}$$

$$P = 19,19,656/-$$

④ $A = 95,50,000$

$P = 20,00,000$

$r = \_\_\_\_\_\ \%$ p.a.c.q.

$n = 8$ years

$\Longrightarrow$ $A = P(1+r)^n$

$$95,50,000 = 20,00,000\left(1+\frac{r}{4}\right)^{32}$$

$$\left(1+\frac{r}{4}\right)^{32} = 4.775$$

$r = 20.0276\ 77864\ \%$ P.a.c.q

⑤ $A = 80,00,000$

$P = 25,00,000$

$r = \_\_\_\_\ \%$ P.a.c.m.

$n = 3$ years

$$A = P(1+r)^n$$

$$80,00,000 = 25,00,000\left(1+\frac{r}{12}\right)^{36}$$

$$\left(1+\frac{r}{12}\right)^{36} = 3.20$$

$$\left(1+\frac{r}{12}\right) = 3.20^{1/36}$$

$r = 39.41\ \%$ P.a.c.m.

⑥ $A = 28,00,000$

$P = ?$

$r = 14\ \%$ P.a.c. monthly $\Longrightarrow$ $A = P(1+r)^n$

$n = 3.75$ years

$$28,00,000 = P\left(1+\frac{0.14}{12}\right)^{45}$$

$$28,00,000 = P\left(1.01166666666\right)^{45}$$

$P = 16,61,396$

⑦

$P = 25,000$

$r = \_\_\_\_\ \%$ P.a.c. semiannually

$n = 17$ years

$A = 2,00,000$

$$\Longrightarrow \quad A = P(1+r)^n$$

$$2,00,000 = 25,000\left(1+\frac{r}{2}\right)^{34}$$

$$\left(1+\frac{r}{2}\right)^{34} = 8$$

$$\left(1+\frac{r}{2}\right) = 8^{1/34}$$

$r = 12.617\%$ p.a.c. semiannually

(8) $A = 2$ crores

$P = 8,00,000$

$r = \_\_\_\_\ \%$ p.a.c. half yearly

$n = 25$ years

$$\Longrightarrow \quad A = P(1+r)^n$$

$$2,00,00,000 = 8,00,000 \times \left(1+\frac{r}{2}\right)^{50}$$

$$\left(1+r/2\right)^{50} = 25$$

$r = 13.304\%$ p.a.c. half yearly

(9) $A = ?$

$P = 30,00,000$

$r = 13\%$ p.a.c. monthly

$n = 2$ years 7 months

$$\Longrightarrow \quad A = P(1+r)^n$$

$$= 30,00,000\left(1+\frac{0.13}{12}\right)^{31}$$

$$= 30,00,000\left(1.010833333333\right)^{31}$$

$A = 41,89,744/-$

$P = 100$, $r = 12\%$ p.a.c.a., $n = 1$ year

$A = ?$

$$\Rightarrow A = P(1+r)^n$$
$$= 100(1+0.12)^1$$
$$= 100(1.12)^1$$
$$= ₹\ 112/-$$

Effectively he has earned 12% interest for the year.

$P = 100$, $r = 12\%$ p.a.c.q., $n = 1$ year

$A = ?$

$$\Rightarrow A = P(1+r)^n$$
$$= 100\left(1+\frac{0.12}{4}\right)^4$$
$$= 100(1.03)^4$$
$$= ₹\ 112.550881$$

Effectively he has earned 12.550881% interest

$P = 100$, $r = 12.550881\%$ p.a.c.a.

$n = 1$ year $A = ?$

$$A = P(1+r)^n$$
$$= 100(1+0.12550881)^1$$
$$= 112.550881$$

Nominal rate of interest

12% p.a.c.q. is Equivalent to 12.550881% p.a.c.a.

Effective rate of interest

- Find effective rate of interest for Nominal rate of interest of 12% p.a.c.q.

$\Rightarrow$

$$\text{Effective rate of interest} = \left(1+\frac{r}{n}\right)^n - 1$$
$$= \left(1+\frac{0.12}{4}\right)^4 - 1$$
$$= 1.03^4 - 1$$
$$= 0.12550881$$

i.e. 12.550881%

where r = Nominal rate of interest

n = No. of conversion periods in a year

- Find effective rate of interest for Nominal rate of 20% p.a.c. semiannually

$\Rightarrow$

$$\left(\begin{matrix}\text{Effective rate}\\ \text{of interest}\end{matrix}\right) = \left(1+\frac{r}{n}\right)^{n} - 1$$

$$= \left(1+\frac{0.20}{2}\right)^{2} - 1$$

$$= 1.10^{2} - 1$$

$$= 0.21 \quad \text{i.e } 21\% \text{ p.a.c.a.}$$

- Find Effective rate of interest for Nominal rate of 21% p.a.c.m.

$\Rightarrow$

$$\text{Effective rate} = \left(1+\frac{r}{n}\right)^{n} - 1$$

$$= \left(1+\frac{0.21}{12}\right)^{12} - 1$$

$$= (1.0175)^{12} - 1$$

$$= 23.143931489\% \text{ p.a.c.a.}$$

$P = 80,000$	$P = 80,000$
$n = 2$ years	$n = 2$ years
$A = ?$	$A = ?$
$r = 21\%$ p.a.c.m.	$r = 23.143931489\%$ p.a.c.a.
$\Rightarrow$	$\Rightarrow$
$A = 80,000\left(1+\frac{0.21}{12}\right)^{24}$	$A = 80,000\left(1+\frac{0.23143931489}{1}\right)^{2}$
$= 80,000 \times 1.0175^{24}$	$= 80,000\left(1.23143931489\right)^{2}$
$= 1,21,315/-$	$= 1,21,315/-$

- $P = 9,00,000$

$r = 13\%$ P.a.c.Q. $\Rightarrow$ $A = P(1+r)^n$

$n = 75$ months

$A = ?$

$= 9,00,000\left(1+\frac{0.13}{4}\right)^{25}$

$= 9,00,000(1.0325)^{25}$

$= 20,02,138/-$

- Find effective rate of interest for nominal rate of 21.505075% P.a.c.Q.

$\Rightarrow$

Effective rate of interest $= \left(1+\frac{r}{n}\right)^n - 1$

$= \left(1+\frac{0.21505075}{4}\right)^4 - 1$

$= (1.053762687 5)^4 - 1$

$= 23.30232525 35\%$ P.a.c.a.

$r = 21.505075\%$ P.a.c.Q.	$r = 23.30232525 35\%$ P.a.c.a.
$P = 1,00,000$	$P = 1,00,000$
$n = 3$ years	$n = 3$ years
$A = ?$	$A = ?$
$\Rightarrow$	$\Rightarrow$
$A = P(1+r)^n$	$A = P(1+r)^n$
$= 1,00,000\left(1+\frac{0.21505075}{4}\right)^{12}$	$= 1,00,000\left(1.2330232525 35\right)^3$
$= 1,00,000(1.053762687 5)^{12}$	$= ₹\ 1,87,462/-$
$= ₹\ 1,87,462/-$	

- Find effective rate for 36.50% p.a.c.daily.

$$\Rightarrow \text{Effective rate} = \left(1+\frac{r}{n}\right)^{n} - 1$$

$$= \left(1+\frac{0.3650}{365}\right)^{365} - 1$$

$$= (1.001)^{365} - 1$$

$$= 44.01877\% \text{ p.a.c.a.}$$

(1) Find effective rate of interest for Nominal rate of 12% p.a.c. monthly

$$\Rightarrow \text{Effective rate} = \left(1+\frac{r}{n}\right)^{n} - 1$$

$$= \left(1+\frac{0.12}{12}\right)^{12} - 1$$

$$= 1.01^{12} - 1$$

$$= 12.68250301\% \text{ p.a.c.a.}$$

$P = 2,00000$	$P = 2,00,000$
$n = 3$ years	$n = 3$ years
$r = 12\%$ p.a.c.m.	$r = 12.68250301\%$ p.a.c.a.
$A = ?$	$A = ?$
$A = P(1+r)^{n}$	$A = P(1+r)^{n}$
$= 2,00,000(1.01)^{36}$	$= 2,00,000(1.12682503)^{3}$
$= 2,86,154/-$	$= 2,86,154/-$

② Nominal rate of r % p.a.c.q is equivalent to 14% effective. Find r.

$\Longrightarrow$

$$\text{Effective rate} = \left(1+\frac{r}{n}\right)^{n} - 1$$

$$0.14 = \left(1+\frac{r}{4}\right)^{4} - 1$$

$$r = 13.3197\% \text{ p.a.c.q.}$$

③ Effective rate of 20% is equivalent to ____ % p.a.c. monthly.

$\Longrightarrow$

$$\text{Effective rate} = \left(1+\frac{r}{n}\right)^{n} - 1$$

$$0.20 = \left(1+\frac{r}{12}\right)^{12} - 1$$

$$\left(1+r/12\right)^{12} = 1.20$$

$$1+r/12 = 1.20^{1/12}$$

$$r = 18.37\% \text{ p.a.c.m.}$$

④ Effective rate of 24.24% p.a.c.a. is equivalent to ____ % p.a.c. weekly

$\Longrightarrow$

$$\text{Effective rate} = \left(1+r/n\right)^{n} - 1$$

$$0.2424 = \left(1+\frac{r}{52}\right)^{52} - 1$$

$$\left(1+r/52\right)^{52} = 1.2424$$

$$1+r/52 = 1.2424^{1/52}$$

$$\therefore \quad r = 21.75\% \text{ p.a.c.weekly}$$

(5)

Nominal rate	Effective rate
18.2436 % P.a.c.m.	$\left(1+\frac{0.182436}{12}\right)^{12}-1$ $= 1.015203^{12}-1 = 19.8491\%$
28 % P.a.c. daily	$\left(1+\frac{0.28}{365}\right)^{365}-1$ $= 1.000767123 28^{365}-1 = 32.2947\%$
32 % P.a.c. ~~for~~ fortnightly	$\left(1+\frac{0.32}{24}\right)^{24}-1$ $= 1.013333333^{24}-1$ $= 37.42188\%$
15.28572819 % P.a.c.q.	$\left(1+\frac{0.15285728 19}{4}\right)^{4}-1$ $= 1.0382\ 14320 47^{4}-1$ $= 16.1845\%$
28.24 % P.a.c.a.	$\left(1+\frac{0.2824}{1}\right)^{1}-1$ $= 28.24\%$
15.932158 % P.a. c. half yearly	$\left(1+\frac{0.15932158}{2}\right)^{2}-1$ $= 1.07966079^{2}-1 = 16.5667\%$
59.2816 % P.a.c.m.	$\left(1+\frac{0.592816}{12}\right)^{12}-1$ $= 1.049401333 33^{12}-1 = 78.361\%$
16.28 % P.a.c.q	$\left(1+\frac{0.1628}{4}\right)^{4}-1$ $= 1.0407^{4}-1 = 17.301136\%$

(6) ____ r ____ % P.a.c.monthly is equivalent to 20.20% P.a.c.q.

$\Rightarrow$

$$\left(1+\frac{r}{12}\right)^{12}-1 = \left(1+\frac{0.2020}{4}\right)^{4}-1$$

$$\left(1+\frac{r}{12}\right)^{12} = 1.0505^{4}$$

$$\left(1+\frac{r}{12}\right)^{12} = 1.217823\,15426$$

$$1+\frac{r}{12} = 1.217823\,15426^{1/12}$$

$$r = 19.86\% \text{ p.a.c.m.}$$

(7) ____ r ____ % P.a.c. semiannually is equivalent to 22.86% P.a.c.m.

$\Rightarrow$

$$\left(1+\frac{r}{2}\right)^{2}-1 = \left(1+\frac{0.2286}{12}\right)^{12}-1$$

$$\left(1+\frac{r}{2}\right)^{2} = 1.01905^{12}$$

$$r = 23.97676\% \text{ p.a.c. semiannually}$$

What is simple annuity ?

⟹

simple annuity is a series of payments/receipts where

- Time Gap between 2 consecutive payments/receipts must be same
- Amount paid/received in every period must be same

. Mr.A invested ₹ 2,00,000 at the end of each year for 5 years in Bank of India @ 15% P.a.
Find amount receivable from BOI at the end of 5 years?

⟹ option ①

Year	Op. Bal.	Interest	Deposit	closing Balance
1	0	0	2,00,000	2,00,000
2	2,00,000	30,000	2,00,000	4,30,000
3	4,30,000	64,500	2,00,000	6,94,500
4	6,94,500	1,04,175	2,00,000	9,98,675
5	9,98,675	1,49,801.25	2,00,000	13,48,476.25

Amount to be received by Mr.A at the end of 5 years = ₹ 13,48,476.25

option ②

Year	Amount receivable at the end of 5 years
1	$2,00,000 \times 1.15^4 = 3,49,801.25$
2	$2,00,000 \times 1.15^3 = 3,04,175$
3	$2,00,000 \times 1.15^2 = 2,64,500$
4	$2,00,000 \times 1.15^1 = 2,30,000$
5	$2,00,000 \times 1.15^0 = 2,00,000$
Total =	13,48,476.25

Amount to be received by mr.A at the end of 5 years = ₹ 13,48,476.25

option ③

$$\text{Future value of annuity Regular} = \text{Periodical amount} \times \left[\frac{(1+r)^n - 1}{r}\right]$$

$$= ₹2,00,000 \left[\frac{1.15^5 - 1}{0.15}\right]$$

$$= ₹13,48,476.25$$

Future value of annuity regular

$$= \text{periodical amount} \times \left[\frac{(1+r)^n - 1}{r}\right]$$

. Mr. X saved ₹5,00,000 at the end of each year for 30 years. Find amount receivable at the end if money is 18% effective.

$\Rightarrow$

Future value of annuity regular

$$= \text{periodical amount} \times \frac{(1+r)^n - 1}{r}$$

$$= ₹5,00,000 \times \left(\frac{1.18^{30} - 1}{0.18}\right)$$

$$= ₹\ 39,54,73,996/-$$

. Mr. A wants to have ₹25 crores in his account at the end of 25 years. Find How much amount he should save at the end of each year if money can fetch 15% ROI?

$\Rightarrow$

$$\text{Future value of annuity regular} = (P.A.) \times \left[\frac{(1+r)^n - 1}{r}\right]$$

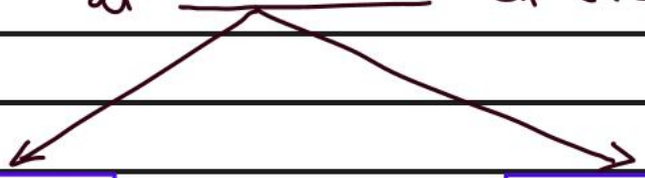
$$25,00,00,000 = P.A. \times \left(\frac{1.15^{25} - 1}{0.15}\right)$$

$$P.A. = 11,74,851/-$$

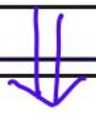
In annuity

If payment/receipt is

at ——— of every period



End	Begining
↓	↓
Annuity regular	Annuity Due
OR	OR
ordinary annuity	Annuity immediate



If type of annuity is not mentioned in the question then it will be treated as **Annuity Regular**

① Mr. X deposited ₹ 90,000 at the end of each year for 55 years. Find amount receivable at the end if he get 13.7575% p.a. interest ?

Future value of annuity regular

$$= P.A. \times \left[\frac{(1+r)^n - 1}{r}\right]$$

$$= 90{,}000 \left(\frac{1.137575^{55} - 1}{0.137575}\right)$$

$$= 78,38,67,841/-$$

② Mr. x deposited ₹5000 at the end of each year for 50 years @ 13% p.a.. Find Future value.

$\Longrightarrow$

Future value of annuity regular $= 5{,}000 \times \left(\frac{1.13^{50}-1}{0.13}\right)$

$= ₹\ 1,72,97,536/-$

③ Mr. M deposited ₹10,000 at the begining of every year for 6 years @ 12% p.a. C.I. Find money receivable at the end of 6 years?

$\Longrightarrow$

option ①

Year	opening Bal	Deposit	Total	Interest	cl. Balance
1	0	10,000	10,000	1,120	11,200
2	11,200	10,000	21,200	2,544	23,744
3	23,744	10,000	33,744	4,049.28	37,793.28
4	37,793.28	10,000	47,793.28	5,735.1936	53,528.4736
5	53,528.4736	10,000	63,528.4736	7,623.417	71,152
6	71,152	10,000	81,152	9738	90,890

Amount receivable at the end of 6 years $= ₹90{,}890$

option-②

Year	Amount receivable at the end of 6 years
1	$10{,}000 \times 1.12^6 = 19{,}738$
2	$10{,}000 \times 1.12^5 = 17{,}623$
3	$10{,}000 \times 1.12^4 = 15{,}735$
4	$10{,}000 \times 1.12^3 = 14{,}050$
5	$10{,}000 \times 1.12^2 = 12{,}544$
6	$10{,}000 \times 1.12^1 = 11{,}200$

Total = 90,890

Amount receivable at the end of 6 years = ₹90,890

option ③

$$\text{Future value of annuity due} = P.A. \times \left[\frac{(1+r)^n - 1}{r}\right] \times (1+r)$$

$$= 10,000 \times \left(\frac{1.12^6 - 1}{0.12}\right) \times 1.12$$

$$= ₹\,90,890$$

Future value of ANNUITY REGULAR	$= \text{periodical amount} \times \left[\frac{(1+r)^n - 1}{r}\right]$
Future value of ANNUITY DUE	$= \text{periodical amount} \times \left[\frac{(1+r)^n - 1}{r}\right] \times (1+r)$

- XYZ ltd. wants ₹ 10 crores for replacement of machinery after 20 years. company decided to keep some amount aside every year which can invested @ 15% P.a.
  Find amount to be kept aside every year?

$\Longrightarrow$ Future value of annuity regular $= P.A. \times \left[\frac{(1+r)^n - 1}{r}\right]$

$$10,00,00,000 = P.A. \times \left(\frac{1.15^{20} - 1}{0.15}\right)$$

periodical amt = 9,76,147/-

- A company wants to create sinking fund of ₹ 25 crores at the end of 10 years for redemption of debentures. Find How much amount should be kept aside at the end of each year if money is 13% effective.

$\Longrightarrow$ Future value of annuity regular $= P.A. \times \boxed{\frac{(1+r)^n - 1}{r}}$

$$25,00,00,000 = P.A. \times \left(\frac{1.13^{10} - 1}{0.13}\right)$$

$$P.A. = 1,35,72,389/-$$

- Mahesh wants buy a palace after 50 years for 50 crores. He decided to keep some amount aside at the end of every year which gets 18.50% interest. Find amount to be kept aside?

$\Longrightarrow$

Future value of annuity regular $=$ periodical amount $\times \boxed{\frac{(1+r)^n - 1}{r}}$

$$50,00,00,000 = P.A. \times \left(\frac{1.1850^{50} - 1}{0.1850}\right)$$

$$P.A. = 19,068/-$$

. Suresh invested ₹ 1,00,000 at the starting of every year for 58 years. Find amount receivable at the end if money can fetch 17.75% interest ?

$\Longrightarrow$

Future value of annuity due $= P.A. \times \left(\frac{(1+r)^n - 1}{r}\right) \times (1+r)$

$= 1,00,000 \left(\frac{1.1775^{58} - 1}{0.1775}\right) \times 1.1775$

$= ₹\ 865,88,93,612/-$

. Find present value of ₹ 2,00,000 payable at the end of each year for 5 years if money is 12% effective ?

$\Longrightarrow$ option ①

money payable at the end of year	present value
1	2,00,000 × 0.892857 = 1,78,571
2	2,00,000 × 0.797193 = 1,59,439
3	2,00,000 × 0.7117802 = 1,42,356
4	2,00,000 × 0.635518 = 1,27,104
5	2,00,000 × 0.567426 = 1,13,485
Total	7,20,955

option ②

present value of annuity regular $=$ (periodical amount × Annuity factor)

$= 2,00,000 \times 3.604776202 28$

$= 7,20,955/-$

(PRESENT VALUE OF ANNUITY REGULAR) = (periodical amount) × (Annuity factor)

(PRESENT VALUE OF ANNUITY DUE) = (periodical amount) × (Annuity factor) × (1+r)

How to find annuity factor on calculator?

$1 \div (1+r)$ then press '=' n times then press GT

$1 \div (1+r)$ then press '= M+' n times then press MRC

- Mr. M took a loan of ₹ 10,00,000 to be repaid in 4 annual equal installments. Find installment amount if rate of interest is 10% p.a.

$\Rightarrow$ (present value of annuity regular) = 10,00,000

periodical amount × Annuity factor = 10,00,000

Installment amount × 3.169865 44634 = 10,00,000

∴ Installment amount = ₹ 3,15,471/-

cross - check

Loan a/c of Mr. M.

Year	opening Balance	Interest	Amount repaid	closing Balance
1	10,00,000	1,00,000	3,15,471	7,84,529
2	7,84,529	78,453	3,15,471	5,47,511
3	5,47,511	54,751	3,15,471	2,86,791
4	2,86,791	28,680	3,15,471	0

- Vinod took a loan of ₹ 25 crores which is to be repaid in 50 annual equal installments. Find installment amount if rate of interest is 12% p.a.

$\Longrightarrow$ present value of annuity regular = 25,00,00,000

Installment amount × Annuity factor = 25,00,00,000

Installment amount × 8.3044984846 = 25,00,00,000

Install. amount = 3,01,04,166/-

- Mr. K took a of ₹ 25,00,000 to buy a car from HDFC Bank. Banking is charging interest of 8.50%. period of repayment is 120 months. Find EMI (Equated monthly installment)

$\Longrightarrow$

P.A. × Annuity factor = 25,00,000

EMI × 80.6544697922 = 25,00,000

EMI = ₹ 30,996/-

- when there are unlimited/infinite periodical payments/receipts : perpetuity

- present value of perpetuity = $\left(\frac{\text{periodical amount}}{r}\right)$

Formulae

① simple interest = $P.n.r$

② Amount when interest is simple = $P(1+nr)$

③ compound interest = $P\left[(1+r)^n - 1\right]$

④ Amount when interest is compound = $P(1+r)^n$

⑤ Future value = present value $\times$ $(1+r)^n$

⑥ present value = Future value $\times$ Discounting factor

⑦ Discounting factor = $1/(1+r)^n$

⑧ Effective rate of interest = $\left(1+\frac{r}{n}\right)^n - 1$

where r = Nominal rate of interest

⑨ Future value of annuity regular = periodical amt $\times \left[\frac{(1+r)^n - 1}{r}\right]$

⑩ Future value of annuity due = periodical amt $\times \left[\frac{(1+r)^n - 1}{r}\right] \times (1+r)$

(11) present value of annuity regular = (periodical amount × Annuity factor)

(12) present value of annuity due = [periodical amount × annuity factor × (1+r)]

(13) present value of perpetuity = $(A/r) = \left(\frac{\text{periodical amt}}{r}\right)$

(14) How to find Discounting factor on calculator for n^{th} year?

⟹ $1 \div (1+r)$ then press '=' till step count comes $(n+2)$

(15) How to find annuity factor on calculator?

⟹ $1 \div (1+r)$ then press '=' till step count comes $(n+2)$ and then press GT

① Nominal rate = 28.2796 %. P.a.c.Q.

Effective rate = ?

$$\Rightarrow \text{Effective rate} = \left(1 + \frac{0.282796}{4}\right)^4 - 1$$

$$= 1.0706994 - 1$$

$$= 31.4225\% \text{ p.a.c.a.}$$

② Effective rate of 15% is equivalent to Nominal rate of ____ %. P.a.c.m.

$$\Rightarrow 0.15 = \left(1 + \frac{r}{12}\right)^{12} - 1$$

$$\left(1 + r/12\right)^{12} = 1.15$$

$$\left(1 + r/12\right) = 1.15^{1/12}$$

$$r = 14.0581\% \text{ p.a.c.m.}$$

③ On a certain sum simple interest at the end of 6.25 years becomes $(3/8)^{th}$ of the sum. Find rate of interest?

$$\Rightarrow SI = P.n.r$$

$$\frac{3}{8}P = P \times 6.25 \times r$$

$$\frac{0.3750}{6.25} = r$$

$$0.06 = r$$

$$\therefore r = 6\% \text{ P.a.S.I.}$$

④ Find amount to be invested to earn monthly simple interest of ₹7200 @ 24% p.a.

$\Rightarrow$ $SI = P \cdot n \cdot r$

$7200 = P \times \frac{1}{12} \times 0.24$

$P = 3,60,000/-$

⑤ A sum invested at SI becomes double in 8 years. Find Number of years sum will becomes 64 times.

$\Rightarrow$

$A = P(1+nr)$	$64\cancel{P} = \cancel{P}(1 + n \times 0.1250)$
$2\cancel{P} = \cancel{P}(1+8r)$	$63 = 0.1250n$
$2 = 1+8r$	$n = 504$ years
$r = 0.1250$	

⑥ A sum invested at CI becomes double in 8 years. Find Number of years sum will becomes 64 times.

$\Rightarrow$

$A = P(1+r)^n$

$2P = P(1+r)^8$

$(1+r)^8 = 2$

$64\cancel{P} = \cancel{P}(1+r)^n$

$2^6 = (1+r)^n$

$((1+r)^8)^6 = (1+r)^n$

$(1+r)^{48} = (1+r)^n \therefore n = 48$ years

	No. of years					
	8	16	24	32	40	48
P	2P	4P	8P	16P	32P	64P

Answer : 48 years

⑦ ⑨ A sum invested at SI becomes triple in 7 years. Find Number of years sum will becomes 81 times.

$\Rightarrow$

$A = P(1+nr)$	$81\cancel{P} = \cancel{P}(1+nr)$
$3\cancel{P} = \cancel{P}(1+7r)$	$81 = 1 + (n \times 2/7)$
$2 = 7r$	$n = 280$ years
$r = 2/7$	

⑥ A sum invested at CI becomes triple in 7 years. Find Number of years sum will becomes 81 times.

$\Rightarrow$

	7	14	21	28
P	3P	9P	27P	81P

Answer : 28 years

(8) Find present value of ₹5,65,843/- to be received after 28 years if money is 18.50% effective.

$\Rightarrow$ present value = Future value × Discounting factor

$= 5,65,843 \times 0.008627522425$

$= 4,882/-$

(9) Find Effective rate of interest for Nominal rate of 28% p.a.c. monthly

$\Rightarrow$ Effective rate $= \left(1+\frac{r}{h}\right)^h - 1$

$= \left(1+\frac{0.28}{12}\right)^{12} - 1$

$= 1.023333333^{12} - 1$

$= 31.8881\%$ p.a.c.a.

(10) Find present value of ₹ 1,38,12,58,052 to be paid after 39 years if money can fetch 18.25% interest.

$\Rightarrow$ present value = Future value × Discounting factor

$= 1,38,12,58,052 \times 0.001447955532$

$= 20,00,000/-$

(11) A House can be purchased by paying ₹ 30 lakhs now and 10 annual equal installments of ₹40,00,000. Find cash down price of House if money is 14% effective.

	present value
30,00,000 payable now	30,00,000
10 equal annual install. of ₹ 40 lakhs each	40,00,000 x 5.216115646 08 = 2,08,64,463
cash down price of House	₹ 2,38,64,463

(12) At what rate of compound interest money will become 8 times in 20 years?

$$A = P(1+r)^n$$

$$8P = P(1+r)^{20}$$

$$(1+r)^{20} = 8$$

$$1+r = 8^{1/20}$$

$$r = 10.9597\% \text{ p.a.c.I.}$$

(13) Mr. A invested ₹ 50,00,000 @ 13% p.a.c.q. for 8.50 years. Find Amount receivable at the end?

$$\text{Future value} = A = P(1+r)^n$$

$$= 50,00,000\left(1+\frac{0.13}{4}\right)^{34}$$

$$= 50,00,000\,(1.0325)^{34}$$

$$= 1,48,33,104/-$$

(14) $A = ?$, $P = 8,19,000$, $r = 16\%$ p.a.c.m., $n = 3Y\,7M$.

$$A = P(1+r)^n$$

$$= 8,19,000 \times \left(1+\frac{0.16}{12}\right)^{43}$$

$$= 8,19,000 \times (1.013333333)^{43}$$

$$= 14,47,552/-$$

(15) $A = 20,000$

$P = 7,000$

$r = $ ——— % p.a.c.m.

$n = 5$ years 6 months

$\Rightarrow A = P(1+r)^n$

$20,000 = 7000\left(1+\frac{r}{12}\right)^{66}$

$2.8571428571 = (1+r/12)^{66}$

$1+r/12 = (2.8571428571)^{1/66}$

$r = 19.2426\%$ p.a.c.m.

(16) A company wants to create a sinking fund of ₹ 55 crores at the end of 40 years. Find amount to be kept aside each year if money can fetch 12.85% interest ?

$\Rightarrow$ Future value of annuity regular $= P.A. \times \left[\frac{(1+r)^n - 1}{r}\right]$

$55,00,00,000 = P.A. \times \left(\frac{1.1285^{40} - 1}{0.1285}\right)$

$P.A. = 5,65,804/-$

Amount to be kept aside at the end of ea year = ₹5,65,804/-

(17) Find EMI = ? If Loan amount is ₹ 90,00,000 @ 10.50% p.a.c.m. (Tenure = 90 months)

$\Rightarrow$ present value of annuity regular = 90,00,000

EMI x Annuity factor = 90,00,000

EMI x 62.1094896278 = 90,00,000

EMI = ₹ 1,44,905/-

(18) Find present value of 15 annual equal installments of ₹ 1,00,000 if money is 14% effective ?

$\Rightarrow$ present value of annuity = P.A. x Annuity factor

= ₹1,00,000 x 6.1421679848 6

= ₹ 6,14,217/-

(19) Find effective rate of Nominal rate of 22 % p.a.c. daily ?

$$\Rightarrow \text{Effective rate} = \left(1+\frac{r}{n}\right)^{n} - 1$$

$$= \left(1+\frac{0.22}{365}\right)^{365} - 1$$

$$= 1.000602739^{365} - 1$$

$$= 24.59\% \text{ p.a.c.a.}$$

(20) 18 % p.a.c.q. is equivalent to ____ %. p.a.c. weekly ?

$$\Rightarrow \left(1+\frac{0.18}{4}\right)^{4} - 1 = \left(1+\frac{r}{52}\right)^{52} - 1$$

$$1.045^{4} = \left(1+\frac{r}{52}\right)^{52}$$

$$1.045 = \left(1+\frac{r}{52}\right)^{13}$$

$$1+\frac{r}{52} = 1.045^{1/13}$$

$$r = 17.6366\% \text{ p.a.c. weekly}$$

(21) 16.5892 % p.a.c. monthly is equivalent to ____ %. p.a.c.q.

$$\Rightarrow \left(1+\frac{0.165892}{12}\right)^{12} - 1 = \left(1+\frac{r}{4}\right)^{4} - 1$$

$$1.013824333333^{12} = \left(1+\frac{r}{4}\right)^{4}$$

$$1.179105\,13003 = \left(1+\frac{r}{4}\right)^{4}$$

$$r = 16.8196\% \text{ p.a.c.q.}$$

(22) A loan of ₹50,000 is to be repaid in 5 equal annual installments of ₹x each. Find x if Rate of interest is 16 %.

$$\Rightarrow \text{Present value of annuity regular} = ₹\,50,000$$

$$x \times \text{Annuity factor} = 50,000$$

$$x \times 3.274293\,65361 = 50,000$$

$$\boxed{x = ₹\,15,270/-}$$

(23) compound interest on certain sum for 2 years @ 10% p.a. is ₹2100. simple interest on same sum at the same rate for 2 years will be:

$\Rightarrow$ $CI = P\left[(1+r)^n - 1\right]$

$2100 = P\left[1.10^2 - 1\right]$

$2100 = P \times 0.21$

$P = 10{,}000$

$SI = P.n.r$

$= 10{,}000 \times 2 \times 10\%$

$= ₹2{,}000/-$

(24) out of certain sum $\frac{1}{3}$rd invested at 3%, $\frac{1}{6}$th invested at 6% at rest is invested at 8% for 2 years. simple interest of 2 years from all these is ₹6000. Find the original sum.

$\Rightarrow$

$\frac{P}{3} \times 3\% \times 2 = 0.02P$

$\frac{P}{6} \times 6\% \times 2 = 0.02P$

$\left(P - \frac{P}{3} - \frac{P}{6}\right) \times 8\% \times 2 = \left(\frac{6P-2P-P}{6}\right) \times 8\% \times 2 = \frac{3P}{6} \times 8\% \times 2 = 0.08P$

Total Interest $= 0.12P = 6{,}000$

$P = 50{,}000/-$

(25) population of a village is 20,000. It increases @ 3% p.a. Find the population after 15 years.

$\Rightarrow$ $A = P(1+r)^n$ is

$= 20{,}000\ (1.03)$

$= 31{,}159/-$

(26) The certain sum of money with simple interest becomes ₹ 2,660 in 3 years and ₹ 3100 in 5 years. Find rate of interest.

$$\Longrightarrow \quad \frac{3100}{2660} = \frac{P(1+5r)}{P(1+3r)}$$

$$3100 + 9300r = 2660 + 13,300r$$

$$440 = 4000r$$

$$r = 0.11 = 11\% \text{ P.a.S.I.}$$

(27) At what rate of compound interest money will become 8 times in 10 years?

$$\Longrightarrow \quad A = P(1+r)^n \qquad \therefore (1+r) = 8^{1/20}$$

$$8P = P(1+r)^{20} \qquad \boxed{r = 10.96\% \text{ P.a.C.I.}}$$

$$\therefore (1+r)^{20} = 8$$

(28) $P = 95,000$, $n = 10$ years

$A = ?$ $r = 8\%$ for first 6 years & 12% for last 4 years. (compound interest)

$$\Longrightarrow$$

$$A = P \times (1+r)^n = 95,000 \times 1.08 \times 1.08 \times 1.08 \times 1.08 \times 1.08 \times 1.08 \times 1.12 \times 1.12 \times 1.12 \times 1.12$$

$$= 95,000 \times (1.08)^6 \times (1.12)^4$$

$$= 95,000 \times 1.586874322944 \times 1.57351936 = 2,37,213/-$$

(29) $P = 98,500$, $n = 15$ years

$r = 16\%$ P.a.C.Q for first 11 years and then 18% P.a.C. semi annually for last 4 years

$A = ?$

$$\Longrightarrow \quad A = 98,500 \times \left(1 + \frac{0.16}{4}\right)^{44} \times \left(1 + \frac{0.18}{2}\right)^{8}$$

$$= 98,500 \times 1.04^{44} \times 1.09^8$$

$$= 98,500 \times 5.61651507789 \times 1.992562664168$$

$$= 11,02,339/-$$

(30) The difference between SI & CI on ₹ 10,00,000 @ 15% p.a. for 12 years is :—

⟹ Compound Interest = $10,00,000\left[(1.15)^{12}-1\right] = 43,50,250/-$

simple interest = $10,00,000 \times 12 \times 15\% = 18,00,000$

Difference = $25,50,250/-$

(31) Discounting factor = ?

(a) $(1+r)^n$ (b) $1/(1+r)$ (c) $1/(1+r)^n$ (d) None

(32) population of a city is 10,00,000. It decreases @ 5% p.a. Find population after 5 years.

⟹ $A = P(1+r)^n$

$= 10,00,000\left(1+\frac{-0.05}{}\right)^5$

$= 10,00,000 \times 0.95^5 = 7,73,781/-$

(33) What sum of money will produce ₹ 28,600 interest in 3 years & 3 months @ 2.50% p.a.S.I.

⟹ simple interest = ₹ 28,600

$P \times n \times r = 28,600$

$P \times 3.25 \times 0.0250 = 28,600$

$\boxed{P = ₹\ 3,52,000}$

(34) Find present value of annuity due of ₹ 3,500 for 10 years @ 12% p.a.

⟹ present value of annuity = P.A. x A factor x $(1+r)$

$= 3,500 \times 5.650223028 \times 1.12$

$= 22,149/-$

(35) The simple interest is $(4/9)^{th}$ of principle and Number of years is equal to rate of interest p.a. Find rate of interest p.a.

$\Longrightarrow$ Simple interest $= P \cdot n \cdot r$

$$\frac{4}{9}P = P \cdot r \times \frac{r}{100}$$

$$\frac{400}{9} = r^2$$

$$r = \frac{20}{3} = 6.666666666\%\ \text{p.a. S.I.}$$

(36) Sachin deposited ₹1,00,000 in a Bank for 2 years at SI. How much interest he will earn. Find Final value of his investment?

$\Longrightarrow$ $SI = P.n.r = 1,00,000 \times 2 \times r = 200,000r$

Final value of his investment $= (1,00,000 + 2,00,000r)$

(37) Rahul invested ₹70,000 in a Bank @ 6.50% p.a. S.I. He received ₹85,925 at the end of term. Find the period for which sum was invested?

$\Longrightarrow$ $A = P(1 + nr)$

$$85,925 = 70,000\ (1 + n \times 0.0650)$$

$$1 + 0.0650n = 1.2275$$

$$0.0650n = 0.2275$$

$$\boxed{n = 3.50 \text{ years}}$$

(38) In what time ₹85,000 will amount to ₹1,57,675 @ 4.50% p.a.s.I.

$\Rightarrow A = P(1+nr)$

$1,57,675 = 85,000\left(1 + n \times 0.0450\right)$

$1 + 0.0450n = 1.855$

$\boxed{n = 19 \text{ years}}$

(39) A sum of money invested at SI amount to ₹6200 in 2 years and ₹7400 in 3 years. The principle and Rate of interest are :

$\Rightarrow P(1+2r) = 6200$

$P(1+3r) = 7400$

$\frac{1+2r}{1+3r} = \frac{62}{74}$

$74 + 148r = 62 + 186r$

$12 = 38r$

$r = \frac{12}{38} = 0.31578947368$ i.e. 31.579%

$P\left(1 + 2 \times 0.31578947368\right) = 6200$

$P \times 1.63157894736 = 6200$

$\boxed{P = 3,800}$

(40) $P = 85,000$, $n = 3$ years, $A = ?$ If

$r = 16\%$ p.a.c.q.

$A = 85000 \times \left(1 + \frac{0.16}{4}\right)^{12}$

$= 85,000 \times (1.04)^{12}$

$= ₹\ 1,36,088/-$

$r = 16\%$ p.a.c.m.

$A = 85,000 \times \left(1 + \frac{0.16}{12}\right)^{36}$

$= 85,000 \times (1.01333333)^{36}$

$= ₹1,36,931/-$

$r = 16\%$ p.a.c.a.

$A = 85,000 \times (1.16)^{3}$

$= ₹1,32,676/-$

(41) On what sum will the compound interest @ 5% p.a. for 2 years will be ₹ 1640 ?

$\Longrightarrow$ compound interest $= P\left[(1+r)^n - 1\right]$

$1640 = P\left[(1.05)^2 - 1\right]$

$P = ₹\ 16{,}000/-$

(42) At what rate of compound interest an investment doubles in 7 years ?

$\Longrightarrow$ $A = P(1+r)^n$

$2P = P(1+r)^7$

$(1+r)^7 = 2$

$1+r = 2^{1/7}$

$\boxed{r = 10.409736997\%\ \text{p.a. c.I.}}$

(43) In what time will ₹8000 amount to ₹8820 @ 10% p.a. c. semiannually

$\Longrightarrow$ $A = P(1+r)^n$

$8820 = 8000\left(1 + \frac{0.10}{2}\right)^{2n}$

$1.1025 = 1.05^{2n}$

$1.05^2 = 1.05^{2n}$

$\therefore \quad 2n = 2$

$n = 1$ year

(44) Find the rate % p.a. if ₹2,00,000 amount to ₹2,31,525 in $1\frac{1}{2}$ years if interest is compounded half yearly?

$\Rightarrow A = P(1+r)^n$

$2,31,525 = 2,00,000\left(1+\frac{r}{2}\right)^3$

$1.157625 = \left(1+\frac{r}{2}\right)^3$

$1+\frac{r}{2} = 1.157625^{1/3}$

$1+\frac{r}{2} = 1.05$

$r = 10\%$ p.a.c. half yearly

(45) A certain sum invested @ 4% p.a. compounded semiannually amount to ₹78030 at the end of one year. Find the sum.

$\Rightarrow A = P(1+r)^n$

$78,030 = P\left(1+\frac{0.04}{2}\right)^2$

$78,030 = P \times 1.02^2$

$P = \left(\frac{78030}{1.0404}\right) = 75,000/-$

$\therefore$ Sum invested = ₹75,000/-

(46) which is better investment of following

~~(a)~~ 12% p.a.c.m. (b) 12.10% p.a.c.q.

$\Rightarrow$

Effective rate $= \left(1+\frac{0.12}{12}\right)^{12} - 1$

$= 1.01^{12} - 1$

$= 12.6825\%$ p.a.c.a.

Effective rate $= \left(1+\frac{0.1210}{4}\right)^4 - 1$

$= 1.030325^4 - 1$

$= 12.66019\%$ p.a.c.a.

$\therefore$ option (a) is better from investment point of view.

(47) A machine depreciated @ 20% on reducing balance. original value of machine is ₹1,00,000. Find its scrap value after 6 years?

$\Rightarrow$

$$A = P(1+r)^n$$

$$= 1,00,000\left[1+(-0.20)\right]^6$$

$$= 1,00,000 \times 0.80^6$$

$$= ₹\,26,214.40$$

(48) population of a town increases by 2% every year. Find Number of years population will increase by 40%.

(a) 7 years (b) 10 years ~~(c)~~ 17 years (approx)

(d) None of these

$\Rightarrow$ original population = P

After 'n' years = 1.40P

$$A = P(1+r)^n$$

$$1.40\cancel{P} = \cancel{P}(1.02)^n$$

$$1.02^n = 1.40$$

$$\text{Log}1.02^n = \text{Log}\,1.40$$

$$n \cdot \text{Log}\,1.02 = \text{Log}\,1.40$$

$$n = \frac{\text{Log}\,1.40}{\text{Log}\,1.02} = \frac{0.1461286563 6}{0.0086000558 6}$$

$$n = 16.9915 \text{ years}$$

(49) The diff bet^n SI & CI is ₹ 110.16 if money invested @ 6% p.a. for 3 years. The principle is:

$\Rightarrow$ CI − SI = 110.16

$$P\left[(1+r)^n - 1\right] - Pnr = 110.16$$

$$P\left[1.06^3 - 1\right] - P \times 3 \times 6\% = 110.16$$

$$0.191016P - 0.18P = 110.16$$

$$0.011016P = 110.16$$

$$\boxed{P = 10{,}000}$$

(50) The annual birth rate and death rate per 1000 are 39.4 and 19.4 resp. Find Number of years population will be doubled ?

~~(a)~~ 35 years (approx) (b) 30 years (approx)

(c) 25 years (approx) (d) None of these

$\Rightarrow$

1000	Add 39.4	deduct 19.40	After Year 1020	2%

$$A = P(1+r)^n$$

$$2\cancel{P} = \cancel{P}(1.02)^n$$

$$1.02^n = 2$$

$$n \log 1.02 = \log 2$$

$$\therefore n = \frac{\log 2}{\log 1.02} = \frac{0.30103312099}{0.0086000\,5586}$$

$$\boxed{n = 35.0036 \text{ years}}$$

Leasing

(51) A company is considering proposal of purchasing a machine either by making a full payment of ₹4,00,000 OR Leasing it by making annual payment of ₹ 1,25,000, which course is better if money can be borrowed @ 14% p.a.c.I. (useful life of machine is 4 years)

$\Rightarrow$

	Buy	Leasing
present value of cash outflow	= 4,00,000	= 1,25,000 × 2.913712304 45 = 3,64,214/−

$\therefore$ Leasing is better.

(52) A person invests ₹500 at the end of each years @ 10% P.a.C.I. Amount standing to his credit one year after he has made his yearly investment for 12th time is :

$\Longrightarrow$

Amount receivable at the end of 12 years $= ₹500 \times \left(\frac{1.10^{12}-1}{0.10}\right) = ₹10,692.142$

Amount standing to his credit after 1 year $= 10,692.142 \times 1.10$

$= ₹11,761/-$

(53) Johnson left ₹1,00,000 with a direction that It should be divided in such a way that his minor sons Tom, Dick, Harry aged 9, 12, 15 years should get equally after attaining age of 25 years. The Rate of interest is 3.50% P.a.C.I. How much each son will get after getting 25 years old ?

(a) ₹50,000 (b) ₹51,947 (c) ₹52,000 (d) None

$\Longrightarrow$

$$\left\{\left[\left(1,00,000 \times 1.035^{10}\right) - 51,947\right] \times 1.035^{3} - 51,947\right\} \times 1.035^{3} - 51,947$$

$= 0$

(54) A Machine wsts ₹5,20,000 with a estimated life of 25 years. A sinking Fund is to be created to replace it by new model at 25% higher wst after 25 years with realisable value of ₹25,000 for old machine. what sum should be kept aside each year if money can fetch 3.50% interest ?

$\Longrightarrow$ sinking Fund to be created $= \left(5,20,000 + 25\%\right) - 25,000$

$= 6,50,000 - 25,000$

$= 6,25,000$

Future value of annuity regular $= 6,25,000$

$$P.A. \times \left[\frac{(1+r)^{n} - 1}{r}\right] = 6,25,000$$

$$P.A. \times \left(\frac{1.0350^{25} - 1}{0.0350}\right) = 6,25,000$$

$P.A. = ₹16,046/-$

Amount to be kept aside at the end of each year $= ₹16046/-$

(55) present value of ₹80,000 due after 10 years @ 16% p.a. is :

$\Rightarrow$ present value = Future value × discounting factor

$= 80,000 \times 0.22668360343$

$= ₹18,135/-$

(56) The diff. between SI & CI on ₹20,00,000 for 18 years @ 6.50% is :

$\Rightarrow$

Diff = CI − SI

$= 20,00,000\left[(1.0650)^{18}-1\right] - (20,00,000 \times 18 \times 6.50\%)$

$= 42,13,309 - 23,40,000$

$= ₹18,73,309/-$

(57) compound interest on half yearly rests on ₹1,00,000 with 9% interest for first 2 years and 10% for next 3 years is :

$\Rightarrow$

$\text{Amount} = 1,00,000 \times (1.045)^4 \times (1.05)^6$

$= 1,00,000 \times 1.1925186\,0062 \times 1.3400956\,4062$

$= ₹1,59,809$

compound interest = ₹1,59,809 − ₹1,00,000

$= ₹59,809/-$